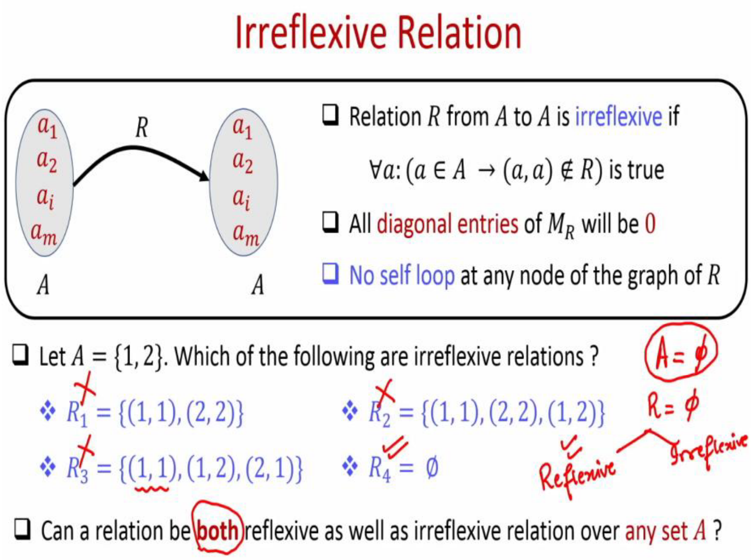

Now let us define another special relation defined from the set to itself which is called the irreflexive relation. And the requirement here is that you need that no element should be related to itself in the relation that means you take any element a from the set A, so this universal quantification over the domain is the set A. You take every element a from the domain or the set A, (a,a) should not be present in the relation

Or the element should not be related to itself. So, it is easy to see that if your relation R is irreflexive, then none of the diagonal entries should be 1 in the relation. So, the matrix for your irreflexive relation will be an n x n matrix. Because the relation is defined from the set A to itself and (a_1, a_1) is not there in the relation, that means the entry number (1, 1) in the matrix will be 0. Similarly (a_2, a_2) is not there in your relation.

That means the entry number (2, 2) in your matrix will be 0 and so on, that means the diagonal entry will be just consisting of 0's or equivalently in terms of the graph representation no self loops will be present, because a_1 will not have any directed edge to itself, a_2 will not have any directed edge to itself and so on. So, again, let me demonstrate irreflexive relations here, so my set A is {1, 2} and I have taken the same 4 relations here.

It turns out that relation R_1 is not irreflexive because you have both (1, 1) and (2, 2) present. Similarly R_2 is not irreflexive, R_3 is also not irreflexive because you have (1, 1) present here, whereas R_4 is a valid irreflexive relation because no element of the form (a, a) is present in R_4. Now it might look that any relation which is reflexive cannot be irreflexive or vice versa but or equivalently can we say that is it possible that I have a relation which is both reflexive as well as irreflexive defined over the same set A.

Well the answer is yes because if you consider the set A equal to the empty set, and if you take the relation R, which is also the empty relation. That is the only relation possible over an empty set A then this relation R is both reflexive as well as irreflexive. It is reflexive because at the first place there is no element present in your set A and hence there is no chance of existence of any (a, a), present in the relation R equal to ϕ.

And due to the same reason since no element is present in the set A you do not need any (a,a) to be present in R. So, the relation R equal to ϕ satisfies the implication, this universal implication given in the definition of reflexive relation as well as irreflexive relation vacuously. So, we can have a relation defined over a set which can be simultaneously reflexive and irreflexive and that can happen in the special case when the set is an empty set.

If A is non empty, then definitely you cannot have a relation which is both reflexive as well as irreflexive.

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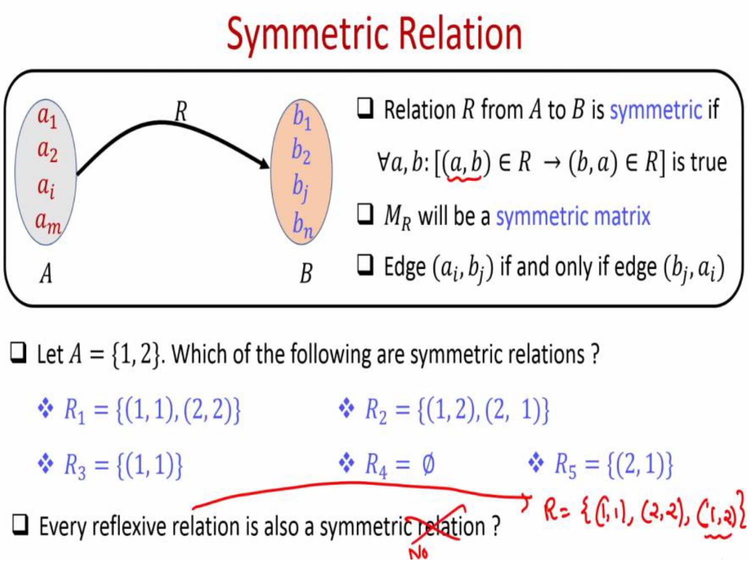

Now let us define symmetric relations, so this relation can be defined from a set A to B where B is might be different from A. So, the relation is from A to B and we say it is symmetric, so as the name suggests symmetric we want here the following to hold, whenever a is related to b as per the relation R, we need that b also should be related to a and that is why the term symmetric here and of course this universal quantification is the domain of a is A and domain of b is B.

I stress here this does not mean that you need every element of the form (a, b) and (b, a) to be present in the relation R, this is an implication. The implication here says that if (a, b) is present in R, then only you need (b, a) to be present in R. If (a, b) is not present at the first place in the relation, then I do not care whether (b, a) is there or not. I do not need (b, a) to be present, so the implication puts the restriction that this condition should be there should be true only if (a, b) is there in the relation.

So, it is easy to see that the matrix for a symmetric relation will always be a symmetric matrix, because if you have a_i R b_j, that means the i, j^{th} entrry will be 1 and since my relation is symmetric, that means I will also have (b_j, a_i) to be present. That means if I take the transpose of M_R, then in the j^{th} row and i^{th} column, the entry will be 1. Equivalently in terms of directed graph representation, if I have a directed edge from the node a_i to b_j and since my relation is symmetric, the edge from b_j to a_i will also be present. So, again let us do this example, I have set A = {1, 2} and I am defining various binary relations from A to A itself. That means in this case my A is equal to B here. Now which of the following relations are symmetric. So, it is easy to see that the first relation is a symmetric relation because this condition is true here.

I can say that since (1, 1) is present in the relation, I also have (1, 1) which can be interpreted as (b, a), also present in the relation. Due to the same reason since (2, 2) is present in the relation which can be interpreted as a being 2 and b being 2, I also have (b, a), present in the relation. Similarly the relation R_2 is a symmetric relation, the relation R_3 is also a symmetric relation because I have (1, 1) present in the relation.

And for symmetric relation (1, 1) also should be present in the relation, which is the case. Turns out that ϕ, is also a symmetric relation here. It satisfies the requirement of symmetric relation because at the first place there is no (a, b) present in my R_4. That means vacuously this implication that this universally quantified statement is true for R_4. And that is why R_4 is also a valid symmetric relation.

But R_5 is not a valid symmetric relation because I have (2, 1) present in my relation but (1, 2) is not present in the relation. So, here is a question for you, can I say that every reflexive relation is also a symmetric relation? So, remember reflexive relation means every element of the form (a, a) will represent in R. And apart from that I might have something additional also present in the relation.

So, if you are given a relation which is reflexive can I say that definitely it is also a symmetric relation and the answer is no. Take the example where A is equal to {1, 2} and let me define a relation R consisting of (1, 1), (2, 2) and say the element (1, 2). This relation is a reflexive relation, but this is not a symmetric relation. But this is not symmetric because you have (1, 2) present in the relation, but you do not have (2, 1) present in the relation.

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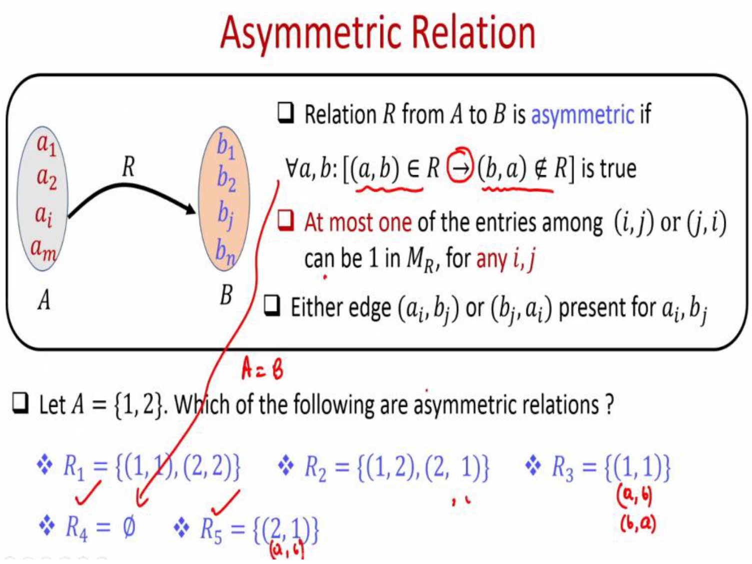

Now the next special relation is the asymmetric relation and the condition here is, if you have a related to b in the relation, then you demand that b should not be related to a. And again this is an implication that means this should hold only if (a, b) is present in the relation at the first place, if (a, b) is not present in the relation, vacuously this statement will be true. So, in terms of matrix notation the property of matrix for an asymmetric relation will be as follows.

You take any i, j^{th} entry, i^{th} row and j^{th} column, you can have at most one of the entries i, j or j, i being 1 in the matrix. You cannot have both entry number i, j 1 as well as j, i also 1. Because that will mean that you have (a_i, b_j) present in R, and (b_j, a_i) also present in the R, which goes against the definition of asymmetric relation. This automatically means that the diagonal entries will be 0.

Because if you have (a, a) present in the R, then that violates the universal quantification here, that serves as a counter example because you have (a, a) present in the R and this (a, a) can be treated as again (a, a) with a and b so here a only is playing the role of both a as well as b. So, you have (a, b) as well as (b, a) both present in this relation R and that serves as a counter example for this universal quantification and hence your relation will not be asymmetric.

So, none of the diagonal entries will be 1. In terms of graph representation, if you take any pair of nodes (a_i, b_j) then either you can have at most one edge, that means you can have either the edge from a_i to b_j or from b_j to a_i or no edge between a_i or b_j. So, this is a wrong statement here, so either edge a_i to b_j or no edge, that is also fine. Because if at the first place there is no relationship between a_i and b_j then that vacuously satisfies this universal quantification.

So, again, here I am taking A and B to be the same sets and I have given you some relations. So, let us see which of these relations are asymmetric. The first relation is not asymmetric because you have (a, b) as well as (b, a), only (1, 1) being present in this relation which serves as both (a,b) as well as (b, a). Due to the same reason R_2 is also not an asymmetric relation because you have both (a, b) as well as (b, a) present here.

Some a and b is there for which this universal quantification is not true. Your relation R_3 is also not asymmetric because you have (a, b) here written as well as (b, a) also present. Whereas the relation R_4 is an asymmetric relation over the set A, because at the first place there is no (a, b) present in this relation R_4, so R_4 vacuously satisfies this universal quantification and R_5 is also an asymmetric relation because you have only (a, b) present in this relation but no (b, a).

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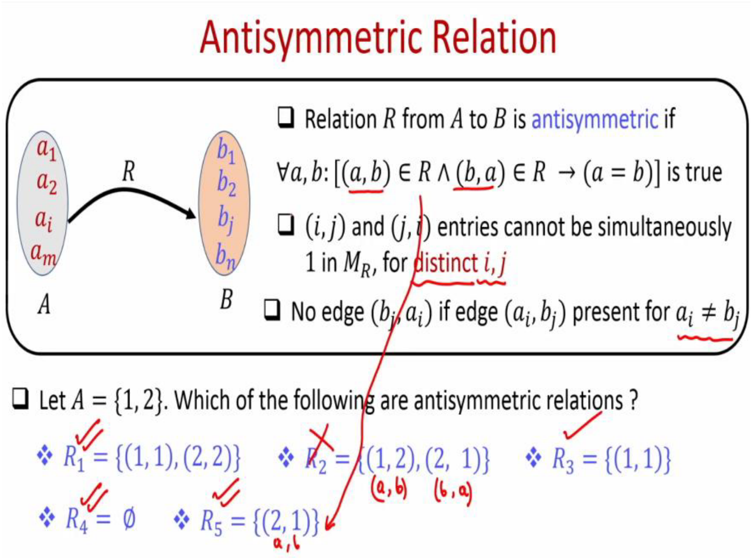

The next special relation is antisymmetric relation and the requirement here is the following. You want that if both (a, b) and (b, a) are present in your relation, that means if you have a case where an element a is related to b and b is also related to a, then that is possible only if a is equal to b. Contra-positively if a is not equal to b, then you can have either (a, b) present in the relation or (b, a) present in the relation or none of them being present in the relation.

That means for distinct elements, you cannot have simultaneously a R b as well as b R a. That is what is the interpretation of this condition. So, in terms of matrix properties if you focus on i^{th} row and j^{th} column where i and j are distinct, then only one of those entries can be 1. Of course both of them can be 0, that is also fine, because that means that neither a R b or nor b R a.

The condition demands that if at all a and b and b and a are both present in the relation, then that is possible only when a and b are same, if they are different and you cannot have both (a, b) as well as (b, a) present in your relation. In terms of graph theoretic properties if you have 2 distinct nodes a_i and b_j, then you cannot have an edge simultaneously from a_i to b_j as well as from b_j to a_i, that is not allowed.

Well, it is fine if you have no edge between these two nodes, that satisfies, that does not violate this universal quantification. So, here are some examples, relation R_1 is an antisymmetric relation because you have (a, b) present here namely (1, 1) and you also have (b, a), present here namely (1, 1), but the implication should be that 1 equal to 1 which is true, same holds for the element (2, 2).

So, this is an example of an antisymmetric relation. But R_2 is not an example of antisymmetric relation because you have a case here namely you have distinct (a, b) such that both (a, b) as well as (b, a) are present in your relation. R_3 is an example of an antisymmetric relation and R_4 is also an example of an antisymmetric relation because it satisfies this universal quantification vacuously.

R_5 is also an example of universal quantification, because you have (a, b) present here, but the (b, a) is not present in the relation R_5, that means the premise of this implication is vacuously true for R_5 and that is why this R_5 is not violating this universal quantification.

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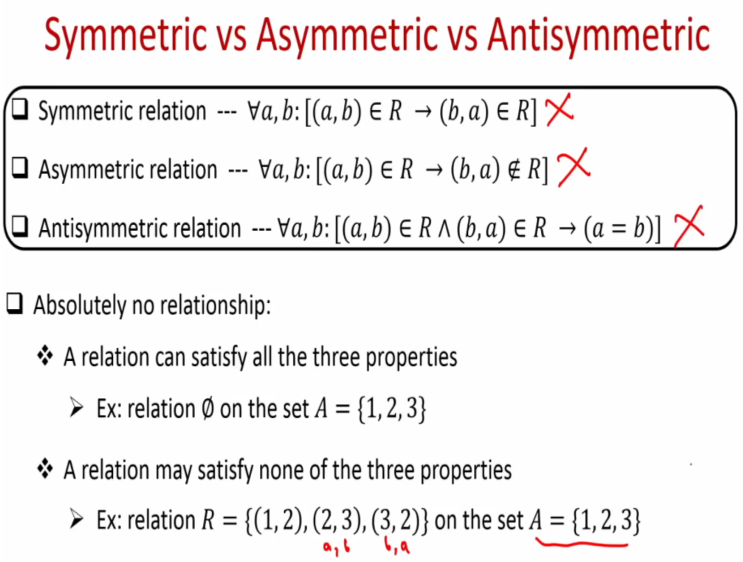

So, we have symmetric relation, asymmetric relation and antisymmetric relation. These are the definitions here and people often wonder that there is some relationship among these three different notions here, this some people think that something which is not symmetric will be asymmetric and similarly they try to conclude some relationship between the symmetric property, asymmetric property and antisymmetric property. But it turns out there is absolutely no relationship. You might have a possibility where you have a relation which satisfies all the three properties, namely if I take the set A and I take the relation R to be empty, then the relation R, which is an empty set here is symmetric as well as asymmetric as well as antisymmetric, it satisfies all these 3 universal quantifications.

Whereas there might be a possibility that a relation satisfies none of these 3 properties. So, if I take the set A to be {1, 2, 3}, and I take this relation R then it is not so, let us see which of these three properties it satisfies, so it is not symmetric because you have (a, b) here, but no (b, a). Only 1 is related to 2, but 2 is not related to 1, so it is not symmetric. It is not an asymmetric relation because you have (a, b) here and simultaneously (b, a) also, then this (2, 3) is there as well as (3, 2) is there.

So, it violates the universal quantification for asymmetric relations and it is not an asymmetric relation because you have (a, b) as well as (b, a) both being present here, even though your a and b are different. Then 2 is not equal to 3 but still you have (2, 3) as well as (3, 2) present in the relation. So, there is no absolute relationship among the notion of symmetric, asymmetric and antisymmetric relations.

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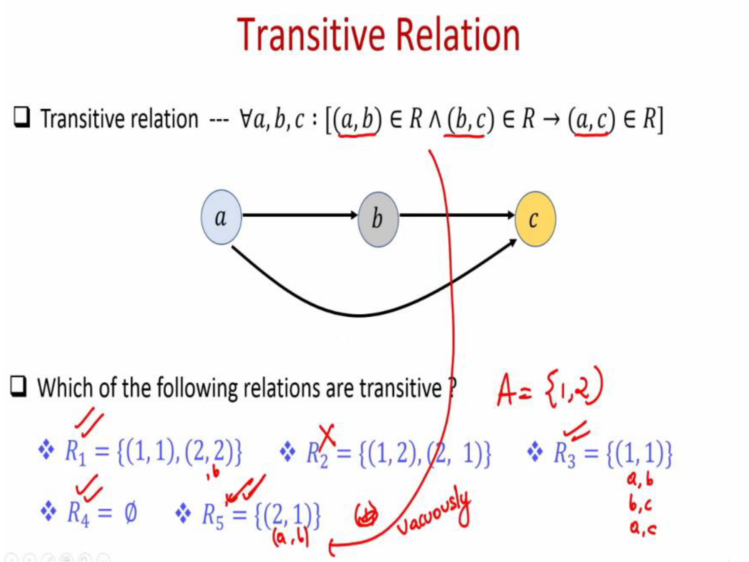

Now let us see the last important relation here, which is the transitive relation. And what do we mean by a transitive relation here, so a relation R is called a transitive relation if the following universal quantification is true. We want that if at all a R b and b R c in your relation, then a also should be related to c. In terms of graph theoretic properties, if you have an edge from a to b in the graph of your relation R.

And if you have; a directed edge from the node b to the node c in the graph of your relation R. Then we need that there should be an edge from a to c as well. And this should hold for every a, b, c, where the domain of a, b, c are from the sets over which the relation is defined. So, let us take this example, so consider the first relation it is a transitive relation, of course, so here everything is defined over a set say {1, 2} and a relation R_1 is transitive.

Because you have; (1, 1) present which can be also considered as (a, b) as well as (b, c) as well as (a, c).
So, again the same is true for (2, 2). But your relation R_2 is not transitive because you have a case here where you have (a, b) present, you also have (b, c) present but no (a, c) is present here. Namely (1, 1) is not present in your relation. Your relation R_3 is also a transitive relation because you have (a, b) present, (b, c) present and you also have corresponding, (a, c) present.

Your R_4 is a transitive relation because it vacuously satisfies this implication because at the first place there is no (a, b) and (b, c) present in your R_4. And your relation R_5 also satisfies vacuously this universal quantification, because you have (a, b) present but there is no (b, c) present that means there is nothing of the form (1, 2) here or (1, 1) here. That means vacuously this condition is true for R_5.

And that is why R_5 is also a transitive relation. So, that brings me to the end of this lecture. Just to summarize in this lecture we introduced binary relations and some special types of binary relations. We also discussed the 2 representations that we follow to represent any binary relation.